Article

RANKING TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS USING ORTHOCENTER OF CENTROIDS: A NEW APPROACH

Vinod Kumar Vaddadi, Boina Anil Kumar, G Jogarao,

Department of BSH (Mathematics), SOS, Gandhi Institute of Engineering and Technology University, Odisha, Gunupur-765022. vinodvaddadi85@gmail.com

Department of BSH (Mathematics), SOS, Gandhi Institute of Engineering and Technology University, Odisha, Gunupur-765022. anilkumarboina@gmail.com

Department of Mathematics (BS&H), Aditya Institute of Technology and Management, Tekkali, India-532201, jogarao.gunda@gmail.com,

ABSTRACT

In this paper, a ranking method for Trapezoidal Intuitionistic Fuzzy Numbers (TrIFNs) based on the orthocenter of centroids is proposed. The proposed approach effectively handles the uncertainty and vagueness associated with TrIFNs, enabling more accurate and reliable decision-making. The orthocenter of centroids is used to calculate a crisp ranking value, facilitating the comparison of TrIFNs. The method's validity and efficiency are demonstrated through numerical examples and comparative analyses with existing ranking methods. Results show that the proposed approach outperforms existing methods in terms of accuracy. This work contributes to the development of intuitionistic fuzzy decision-making theories and applications, particularly in fields such as multi-criteria decision-making, risk assessment, and optimization.
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Keywords: Trapezoidal Intuitionistic Fuzzy Numbers, Orthocenter of Centroids, Ranking Method, Fuzzy Decision-Making, Uncertainty.

1. Introduction

Traditional fuzzy sets don't account for the degree of hesitancy among set elements or address real-world data inaccuracies. The non-membership functions also having limitations. To address these gaps, Atanassov [3] significantly contributed to fuzzy set theory by introducing the notion of intuitionistic fuzzy sets(IFSs), which combine functions related to membership and non-membership., providing a more comprehensive framework for handling uncertainty.

It is easy to express the vagueness of fuzzy sets in this manner. Fuzz numbers are specialized kinds of fuzzy sets that can be considerably useful to solve real-world linear programming problems that are based on uncertain data, according to [1]. Fuzzy arithmetic, fuzzy relation and fuzzy decision making pose a number of fundamental problems. Before a decision maker can make a decision, fuzzy numbers must be ranked. Real numbers of ranks can be ordered linearly by the relations <,>,=, but the fuzzy numbers do not possess this kind of inequality primarily on account of the point that fuzzy numbers represent probability distributions, making this challenging to ascertain whether what is smaller one or what is larger one than the other. Every single fuzzy number should be mapped to a line of reference in which there is an existence of a natural order. The fuzzy numbers can be ordered in an efficient manner by what is called ranking function. As a result, numerous approaches for ordering ambiguous numbers have been proposed. So as to express uncertainty pertaining to “generalized FN”, the use of IFN is appropriate as noted from available sources in literature. There exists in the field, many approaches for ranking IFNs. Researchers [13] IFNs were divided into two families, each with its own set of criteria, and a ranking algorithm was developed for them. By accepting a statistical outlook, Mitchell [9], (2004) suggested interpreting triangular intuitive fuzzy numbers regarding set of common Fuzzy- numbers so as to resolve issues encountered in decision making process on account of multi-attributes[9]. A new method of ranking system to compare IFN was introduced by Kumar and Kaur [7], until which there were many shortcomings in current rankings in the existing systems. The procedure for evaluating IFN was elaborated in their studies [16,17,18,19] addressed cases wherein the vagueness is bigger than the membership fuzzy number. While on the contrary, Intuitionistic- fuzzy-number possess lower levels of uncertainty as compared to that of membership fuzzy number which is a unique technique combining membership and non-membership functions in relation to fuzzy numbers which are Intuitionistic, and the concept is applied to investigate problem that involve uncertainty.

The structure of this paper is as follows: An introduction to fuzzy concepts, such as intuitionistic trapezoidal fuzzy numbers, is given in Section 2. Section 3 introduces a new method for exploiting the orthocenters of centroids to rank intuitionistic fuzzy numbers. Numerical examples and a comparative analysis of generalised intuitionistic numbers are presented in Section 4. The results and debate are finally covered in Section 5.

2. Preliminaries [1]:

Fuzzy set (FS)[20]: A fuzzy set $\bar{F}$ of universe of discourse U is expressed by a membership function $f_{\bar{F}}: U \to [0,1]$, here $f_{\bar{F}}(x)$ is the degree of an element x in IFS $\bar{F}$ and is denoted by $\bar{F} = \{(x, f_{\bar{F}}(x))/x \in U\}$.

Intuitionistic Fuzzy Set(IFS)[20]: A set $\hat{F} = \{(x, f_{\hat{F}}(x), g_{\hat{F}}(x)) | x \in U\}$ defines an Intuitionistic Fuzzy Set $\hat{F}$ in U, where $f_{\hat{F}}$, and $g_{\hat{F}}$ are functions from U to [0,1] and indicate the element x's degree of membership and non-membership in U respectively, also that satisfies an inequality $0 \le f_{\hat{F}}(x) + g_{\hat{F}}(x) \le 1$, for every $x \in U$.

Intuitionistic Convex Fuzzy Set(IFCS)[20]: A fuzzy Intuitionistic Set $\hat{F} = \{(x, f_{\hat{F}}(x), g_{\hat{F}}(x)) | x \in U\}$ defines an intuitionistic fuzzy convex set $\hat{A}$ in U if non-membership function $g_{\hat{F}}(x)$ is fuzzy concave and membership function $f_{\hat{F}}(x)$ is fuzzy convex. That is $g_{\hat{F}}(\lambda x + (1-\lambda)y) \ge g_{\hat{F}}(x) \wedge (y)$ and $f_{\hat{F}}(\lambda x + (1-\lambda)y) \ge f_{\hat{F}}(x) \wedge f_{\hat{F}}(y)$, where $x, y \in U, \lambda \in [0,1]$.

Intuitionistic Normal Fuzzy Set(IFNS)[13]: A normal fuzzy set $\hat{F}$ is defined as one in which there exist at least two elements, x and y, within the universe of discourse U such that $g_{\hat{F}}(y)$ equals 1 and $f_{\hat{F}}(x)$ equals 1.

Intuitionistic Fuzzy Number(IFN)[13]: An intuitionistic fuzzy set $\hat{F}=\{(x,f_{\hat{F}}(x),g_{\hat{F}}(x))|x\in U\}$ of real line R is called an intuitionistic fuzzy number if it is satisfying the following;

1. $\hat{F}$ has bounded support.
2. $\hat{F}$is intuitionistic fuzzy normal set(IFNS)
3. $f_{\hat{F}}(x)$and$g_{\hat{A}}(x)$ are piecewise continuous and take real values
4. $\hat{F}$is intuitionistic fuzzy convex set(IFCS)

Trapezoidal Intuitionistic Fuzzy Number(TrIFN)[13]:A Trapezoidal Intuitionistic Fuzzy Number(TrIFN) of $\hat{F}$ is characterized by the parameters $\nu_1\leq \mu_1\leq \nu_2\leq\mu_2\leq\mu_3\leq\nu_3\leq\mu_4\leq\nu_4$and is represented as by $F=(\nu_1,\mu_1,\nu_2,\mu_2,\mu_3,\nu_3,\mu_4,\nu_4)$. In this context, we define

$$f_{\hat{F}}(x)=\begin{cases}0, & x<\mu_1\\ \frac{x-\mu_1}{\mu_2-\mu_1}, & \mu_1\leq x\leq\mu_2\\ 1, & \mu_2\leq x\leq\mu_3\\ \frac{x-\mu_4}{\mu_3-\mu_4}, & \mu_3\leq x\leq\mu_4\\ 0, & \mu_4<x\end{cases} \text{ and } \quad g_{\hat{F}}(x)=\begin{cases}1, & x<\nu_1\\ \frac{x-\nu_2}{b_1-\nu_2}, & \nu_1\leq x\leq\nu_2\\ 0, & \nu_2\leq x\leq\nu_3\\ \frac{x-\nu_3}{\nu_4-\nu_3}, & \nu_3\leq x\leq\nu_4\\ 1. & \nu_4<x\end{cases}$$

Given, $\nu_2=\nu_3$and$\mu_2=\mu_3$, then we get a triangular intuitionistic fuzzy number(TrIFN) with parameters $\nu_1\leq\mu_1\leq(\nu_2=\mu_2=\mu_3=\nu_3)\leq\mu_4\leq\nu_4$which is represented by $F=(\nu_1,\mu_1,\nu_2,\mu_4,\nu_4)$.

Trapezoidal Intuitionistic Fuzzy Numbers(TrIFNs) Operations [13]

If $F_1=(\nu_1,\mu_1,\nu_2,\mu_2,\mu_3,\nu_3,\mu_4,\nu_4)$ and $F_2=(\nu_1',\mu_1',\nu_2',\mu_2',\mu_3',\nu_3',\mu_4',\nu_4')$ are two TrIFNs, then we defined addition, subtraction, multiplication, division and inverse as follows respectively,

i. $A \triangle\!\!\!\!+ B=(\nu_1+\nu_1',\mu_1+\mu_1',\nu_2+\nu_2',\mu_2+\mu_2',\mu_3+\mu_3',\nu_3+\nu_3',\mu_4+\mu_4',\nu_4+\nu_4')$

ii. $A \triangle B=(\nu_1-\nu_4',\mu_1-\mu_4',\nu_2-\nu_3',\mu_2-\mu_3',\mu_3-\mu_2',\nu_3-\nu_2',\mu_4-\mu_1',\nu_4-\nu_1')$

iii. $A \triangle\!\!\!\!\times B=(\nu_1\nu_1',\mu_1\mu_1',\nu_2\nu_2',\mu_2\mu_2',\mu_3\mu_3',\nu_3\nu_3',\mu_4\mu_4',\nu_4\nu_4')$

iv. $A \ominus B=(\nu_1\div\nu_4',\mu_1\div\mu_4',\nu_2\div\nu_3',\mu_2\div\mu_3',\mu_3\div\mu_2',\nu_3\div\nu_2',\mu_4\div\mu_1',\nu_4\div\nu_1')$, where $\nu_1'\neq0,\mu_1'\neq0,\nu_2'\neq0,\mu_2'\neq0,\mu_3'\neq0,\nu_3'\neq0,\mu_4'\neq0,\nu_4'\neq0$.

v. $A^{-1}=(1\div\nu_4,1\div\mu_4,1\div\nu_3,1\div\nu_3,1\div\mu_2,1\div\nu_2,1\div\mu_1 1\div\nu_1,)$ Where$\nu_1\neq0,\mu_1\neq0,\nu_2\neq0,\mu_2\neq0,\mu_3\neq0,\nu_3\neq0,\mu_4\neq0,\nu_4\neq0$.

2. Ranking Intuitionistic Fuzzy Numbers(IFNs)through the Orthocenters of Centroids-Based Method

The trapezium$f_{\hat{F}}(x)$divides into three triangles ΔABE, ΔBEC, and ΔECD. The orthocenterO_f of centroidsG_1,G_2,and G_3 of these three triangles is chosen as the required reference point. Similarly, we also derive one more orthocenter fromt he trapezium$g_{\hat{F}}(x)$. The purpose of choosing these reference points orthocenters is to calculate Euclidean distance between these two centers and then we derive ranking function to solve transporting problems by using these ranking functions.

Let $F_1=(\nu_1,\mu_1,\nu_2,\mu_2,\mu_3,\nu_3,\mu_4,\nu_4;f_{\hat{F}},g_{\hat{F}})$is a trapezoidal intuitionistic fuzzy number (TrIFN) which represents the figure 1.

Let G_1 be the centroid of ΔABE. Then $G_1 = \left(\frac{3\mu_1+2\mu_2+\mu_4}{6}, \ \frac{1}{3}\right)$.

Let G_2 be the centroid of ΔBEC. Then $G_2 = \left(\frac{\mu_1+2\mu_2+2\mu_3+\mu_4}{6}, \frac{2}{3}\right)$.

Let G_3 be the centroid of ΔECD. Then $G_3 = \left(\frac{\mu_1+2\mu_3+3\mu_4}{6}, \ \frac{1}{3}\right)$.

Let O_f be the orthocenter of the triangle $\Delta G_1G_2G_3$. Then slope of $G_2G_3 = \frac{1}{\mu_2-\mu_4}$ and slope of $G_1P \perp$ slope of G_2G_3. Therefore, the slope of $G_1P = \mu_4 - \mu_2$.

The line $\overline{G_1P}$ passing through the point $G_1 = \left(\frac{3\mu_1+2\mu_2+\mu_4}{6}, \ \frac{1}{3}\right)$ with the slope $\mu_4 - \mu_2$. Then the equation of the line is $\left(y - \frac{1}{3}\right) = (\mu_4 - \mu_2)\left(x - \frac{3\mu_1+2\mu_2+\mu_4}{6}\right)$ implies

$$y = x(\mu_4 - \mu_2) - \frac{(\mu_4-\mu_2)(3\mu_1+2\mu_2+\mu_4)}{6} + \frac{1}{3}. \qquad \textbf{(1)}$$

Now, the slope of $G_1G_2 = \frac{1}{\mu_3-\mu_1}$. Then slope of $G_1G_2 \perp$ slope of G_3Q. Therefore, the slope of $G_3Q = \mu_1 - \mu_3$. The line $\overline{G_3Q}$ passing through the point $G_3 = \left(\frac{\mu_1+2\mu_3+3\mu_4}{6}, \ \frac{1}{3}\right)$ with the slope $\mu_1 - \mu_3$. Then the equation of the line is $y - \frac{1}{3} = (\mu_1 - \mu_3)\left(x - \frac{\mu_1+2\mu_3+3\mu_4}{6}\right)$ implies

$$y = (\mu_1 - \mu_3)\left(x - \frac{\mu_1+2\mu_3+3\mu_4}{6}\right) + \frac{1}{3}. \qquad \textbf{(2)}$$

By solving equations (1) &(2), we obtain the orthocenter O_f of the triangle $\Delta G_1G_2G_3$. Therefore

$$O_f = O(x_f, y_f)$$

$$= \left(\frac{(\mu_1-\mu_3)(\mu_1+2\mu_3+3\mu_4)-(\mu_4-\mu_2)(3\mu_1+2\mu_2+\mu_4)}{6(\mu_1+\mu_2-\mu_3-\mu_4)}, \qquad \frac{(\mu_3-\mu_1)(\mu_2-\mu_4)+1}{3}\right),$$

$$O_f = O(x_f, y_f) = \left(\frac{\mu_1+2\mu_3+2\mu_2+\mu_4}{6}, \qquad \frac{(\mu_3-\mu_1)(\mu_2-\mu_4)+1}{3}\right)$$

The trapezium $g_{\hat{F}}(x)$ divides into three triangles $\Delta A'B'E'$, $\Delta B'E'C'$, and $\Delta E'C'D'$. The orthocenter O_g of centroids G_1', G_2', and G_3' of the above three triangles respectively is chosen as the required reference point.

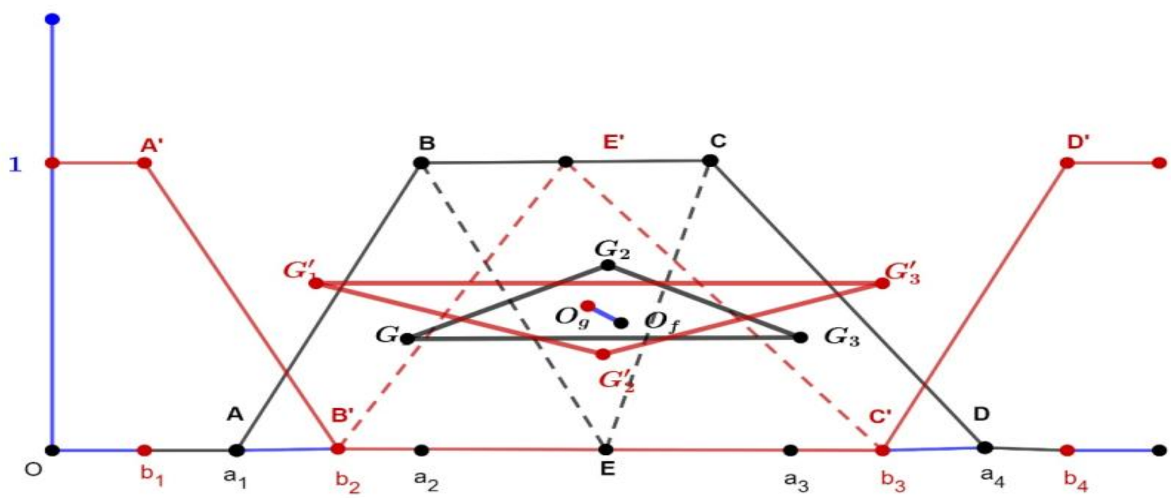

Figure (a) : Trapezoidal Intuitionistic Fuzzy Number(TrIFN)

The orthocenter of centroids of non-membership triangles:

Let G_1' be the centroid of $\Delta A'B'E'$. Then $G_1' = \left(\frac{3\nu_1+2\nu_2+\nu_4}{6}, \frac{2}{3}\right)$

Let G_2' be the centroid of $\Delta B'E'C'$ is $G_2' = \left(\frac{\nu_1+2\nu_2+2\nu_3+\nu_4}{6}, \frac{1}{3}\right)$

Let G_3' be the centroid of$\Delta E'C'D'$ is $G_3' = \left(\frac{\nu_1+2\nu_3+3\nu_4}{6}, \frac{2}{3}\right)$

Let's we derive the orthocenter $O_g = (x_g, y_g)$ of the triangle consisting of the vertices G_1', G_2', G_3' of the non-membership function of the trapezoidal intuitionistic fuzzy number(TrIFN)$F_1 = (\nu_1, \mu_1, \nu_2, \mu_2, \mu_3, \nu_3, \mu_4, \nu_4)$ as follows;

The slope of $G_1'G_2' = \frac{-1}{-\nu_1+\nu_3}$and slope of$G_3'P'$is perpendicular to slope of$G_1'G_2'$. Then the slope of $G_1'G_2'$ is $\nu_3 - \nu_1$. The line $\overline{G_3'P'}$passing through the point$G_3' = \left(\frac{\nu_1+\nu_3+3\nu_4}{6}, \frac{2}{3}\right)$with the slope $\frac{\nu_3-2\nu_1}{2}$is

$$y = \frac{2}{3} + (\nu_3 - \nu_1)x - \frac{(\nu_3-\nu_1)(\nu_1+2\nu_3+3\nu_4)}{6}. \quad \textbf{(3)}$$

The slope of $G_2'G_3'$ is $\frac{1}{-\nu_2+\nu_4}$and slope of $G_2'G_3'$ is perpendicular to $G_1'Q'$. Therefore, the slope of $G_1'Q'$ is $(\nu_2 - \nu_4)$.The line $\overline{G_1{}'Q'}$passing through the point $G_1' = \left(\frac{3\nu_1+2\nu_2+\nu_4}{6}, \frac{2}{3}\right)$with the slope$(\nu_2 - \nu_4)$is

$$y = \frac{2}{3} + (\nu_2 - \nu_4)x - \frac{(\nu_2-\nu_4)(3\nu_1+2\nu_2+\nu_4)}{6} \textbf{(4)}$$

By solving Equations (3)&(4),we obtain the orthocenter of the triangle$G_1'G_2'G_3'$.

Therefore, orthocenter of $\Delta G_1'G_2'G_3'$ is

$$O_g = O(x_g, y_g) = \left(\frac{(\nu_3-\nu_1)(\nu_1+2\nu_3+3\nu_4) - (\nu_2-\nu_4)(3\nu_1+2\nu_2+\nu_4)}{6(\nu_3-\nu_1-\nu_2+\nu_4)}, \frac{2}{3} + \frac{(\nu_3-\nu_1)(\nu_2-\nu_4)}{3}\right)$$

$$O_g = O(x_g, y_g) = \left(\frac{\nu_1+2\nu_2+2\nu_3+\nu_4}{6}, \frac{2}{3} + \frac{(\nu_3-\nu_1)(\nu_2-\nu_4)}{3}\right).$$

3. Euclidian distance between the orthocenters of centroids of trapezoidal intuitionistic fuzzy number:

(A novel way to rank trapezoidal intuitionistic fuzzy numbers (TrIFNs)

We define a ranking function as the distance between the orthocenters of centroids of trapezoidal intuitionistic fuzzy number (TrIFN) which is the Euclidian distance. For a trapezoidal intuitionistic fuzzy number $F_1 = (\nu_1, \mu_1, \nu_2, \mu_2, \mu_3, \nu_3, \mu_4, \nu_4; f_{\hat{A}}, g_{\hat{A}})$, we derive $Rank(F_1) = \left[\left(x_f(F_1) - x_g(F_1)\right)^2 + \left(y_f(F_1) - y_g(F_1)\right)^2\right]^{1/2}$ which is the Euclidian distance between the two orthocenters. Therefore,

$$Rank(F_1) = \left[\left(\frac{\mu_1+2\mu_3+2\mu_2+\mu_4}{6} - \frac{\nu_1+2\nu_2+2\nu_3+\nu_4}{6}\right)^2 + \left(\frac{(\mu_3-\mu_1)(\mu_2-\mu_4)+1}{3} - \frac{2+(\nu_3-\nu_1)(\nu_2-\nu_4)}{3}\right)^2\right]^{1/2}.$$

Characteristics of the trapezoidal intuitionistic fuzzy number's ranking function:

i. $F_1 \preccurlyeq F_2$, if $Rank(F_1) \leq Rank(F_2)$

ii. $F_1 \approx F_2$, if $Rank(F_1) = Rank(F_2)$

iii. $F_1 \succcurlyeq F_2$, if $Rank(F_1) \geq Rank(F_2)$

iv. $Rank(F_1) = 0$ if and only if two orthocenters are coincided

v. $Rank(kF_1) \neq k\ Rank(F_1)$ where k is a constant.

Numerical Examples:

i. If $F_1 = (0.5,0.7,0.9,1.1,1.3,1.5,1.7,1.9)$ and $F_2 = (0.1,0.3,0.5,0.7,0.9,1.1,1.3,1.5)$,

S. N	Fuzzy number	Wang	L Popa	P K De	Rasvani	Suresh Mohan	K Arun Kumar	S K Bharathi	Viswanadham	Centroid ofcenters	Proposed method
1	F_1=(-1, 1, 2,4,-2,0,3,3)	1.2	1.5	3.833333	1.5	1.5	0.487878	0.45	2.666667	0.592593	5
	F_2=(-3,0,5,4,-4,-1,5,6)	1.2	1.5	7.5	1.833333	0.581512	0.581512	0.333333	5.169354	0.814815	10.00139
2	F_1=(0.4,0.8,.11,13,0.4,0.8,.11,13)	5.04	6.3	17.16667	6.166667	6.166667	1.49092	3.25	0	2.701235	0.333333
	F_2=(0.11,0.3,0.4,0.6,0.11,0.3,0.4,0.6)	0.282	0.3525	0.581667	0.351667	0.265427	0.265427	0.2775	0	0.156049	0.333333
3	F_1=(0.5,0.7,0.9,1.1,1.3,1.5,1.7,1.9)	0.64	0.8	1.133333	0.8	1.2	3.349046	0.9	0.8	0.533333	0.866667
	F_2=(0.1,0.3,0.5,0.7,0.9,1.1,1.3,1.5)	0.32	0.4	0.733333	0.4	2.077525	2.077525	0.5	0.8	0.355556	0.866667
4	F_1=(0.2,0.3,0.4,0.5,0.1,0.3,0.4,0.6)	0.28	0.35	0.516667	0.35	0.341667	0.232725	0.191667	0.016997	0.153086	0.327092
	F_2=(0.1,0.2,0.3,0.4,0,0.2,0.3,0.5)	0.26	0.325	0.566667	0.3	0.168677	0.168677	0.125	0.050028	0.118519	0.340359
5	F_1=3.6301, 4.7371, 5.597, 3.0232, 4.1301,6.3049,7.3049)	4.15382	5.192275	6.8154	5.183867	5.191775	3.259737	2.809742	1.058716	2.25883	1.783932
	F_2=(2.4072, 3.7202, 4.9934, 6.0332, 1.76, 2.9072, 5.5332, 6.5332)	3.4308	4.2885	6.368733	4.311267	1.750787	1.750787	2.533852	1.288487	1.845516	2.236237
6	F_1=(0.45003, 5.4406, 6.5488, 7.3417, 3.726, 5.0003, 6.8417, 7.6346)	3.956226	4.945283	8.331112	5.295088	5.567928	6.152877	9.622962	0.785089	2.505245	1.560729
	F_2=(3.2526, 4.4605, 5.5236, 6.6684, 2.6066, 3.7526, 6.1684, 7.1684)	3.98102	4.976275	6.828867	4.981533	2.653278	2.653278	2.820833	1.192906	2.156415	2.051254
7	F_1=(2.68, 3,3,3.71,2.2,3,3,4.67)	4.15382	5.192275	6.8154	5.183867	5.191775	3.259737	2.809742	1.058716	1.364444	0.087837
	F_2=(2.75,6,6,9.375,2.38,6,6,16.2)	3.98102	4.976275	6.828867	4.981533	2.653278	2.653278	2.820833	1.192906	2.832222	8.387698

$$Rank(F_1) = \left[\left(\frac{\mu_1 + 2\mu_3 + 2\mu_2 + \mu_4}{6} - \frac{\nu_1 + 2\nu_2 + 2\nu_3 + \nu_4}{6}\right)^2 + \left(\frac{(\mu_3 - \mu_1)(\mu_2 - \mu_4) + 1}{3} - \frac{2 + (\nu_3 - \nu_1)(\nu_2 - \nu_4)}{3}\right)^2\right]^{1/2}$$

Therefore, $Rank(F_1) = Rank(F_2) = 0.866667$. Therefore, $F_1 = F_2$.

ii. If $F_1 = (-1, 1, 2,4, -2,0,3,3)$ and $F_2 = (-3,0,5,4, -4, -1,5,6)$, then $Rank(F_1) = 5$ and $Rank(F_2) = 10.00139$. Therefore, $F_1 < F_2$.

iii. If $F_1 = (0.2,0.3,0.4,0.5,0.1,0.3,0.4,0.6)$ and $F_2 = (0.1,0.2,0.3,0.4,0,0.2,0.3,0.5)$, then $Rank(F_1) = 0.32709156$ *and* $Rank(F_2) = 0.340359287$. Therefore, $F_1 > F_2$.

iv. Let $F_1 = (-1,1,2,4,-2,0,3,3)$ and $F_2 = (-3,0,5,4,-4,-1,5,6)$ are two Trapezoidal Intuitionistic fuzzy numbers, then, $F_1 + F_2 = (-1-3, 1+0, 2+5, 4+4, -2-4, 0-1, 3+5, 3+6) = (-4,1,7,8,-6,-1,8,9)$.
$Rank(F_1) = 2.357023$, $Rank(F_2) = 10.00139$ and $Rank(F_1 + F_2) = 11.33456$.

v. Let $F_1 = (0.4,0.8,0.8,0.13,0.2,0.8,0.8,0.15)$ and $F_2 = (0.11,0.3,0.3,0.6,0.1,0.3,0.3,0.7)$. Then $F_1 + F_2 = (0.51,1.1,1.1,0.73,0.3,1.1,1.1,0.85)$ and $F_1 - F_2 = (0.29,0.5,0.5,-0.47,0.1,0.5,0.5,-0.55)$. Therefore $Rank(F_1 + F_2) = 0.340833333$ and $Rank(F_1 - F_2) = 0.175833333$.

4. Comparative Analysis

S. No.	Fuzzy number	Wang	L Popa	P K De	Rasvani	Suresh Mohan	K Arun Kumar	SK Bharathi	Viswanadham	Pardha Saradhi	Proposed method
1	F_1=(-1, 1, 2,4,-2,0,3,3) F_2=(-3,0,5,4,-4,-1,5,6)	$F_1 \approx F_2$	$F_1 \approx F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \prec F_2$	$F_1 \prec F_2$
2	F_1=(0.4,0.8,.11,13,0.4,0.8,.11,13) F_2=(0.11,0.3,0.4,0.6,0.11,0.3,0.4,0.6)	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \approx F_2$	$F_1 \succ F_2$	$F_1 \approx F_2$
3	F_1=(0.5,0.7,0.9,1.1,1.3,1.5,1.7,1.9) F_2=(0.1,0.3,0.5,0.7,0.9,1.1,1.3,1.5)	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \approx F_2$	$F_1 \succ F_2$	$F_1 \approx F_2$
4	F_1=(0.2,0.3,0.4,0.5,0.1,0.3,0.4,0.6) F_2=(0.1,0.2,0.3,0.4,0,0.2,0.3,0.5)	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \approx F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$
5	F_1=3.6301, 4.7371, 5.597, 3.0232,4.1301,6.3049,7.3049) F_2=(2.4072, 3.7202, 4.9934, 6.0332, 1.76, 2.9072, 5.5332, 6.5332)	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$
6	F_1=(0.45003, 5.4406, 6.5488, 7.3417, 3.726, 5.0003, 6.8417, 7.6346) F_2=(3.2526, 4.4605, 5.5236, 6.6684, 2.6066, 3.7526, 6.1684, 7.1684)	$F_1 \prec F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$
7	F_1=(2.68,3,3,3.71,2.2,3,3,4.67) F_2=(2.75,6,6,9.375,2.38,6,6,16.2)	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \succ F_2$	$F_1 \prec F_2$	$F_1 \prec F_2$	$F_1 \prec F_2$	$F_1 \prec F_2$

The comparative study of Intuitionistic fuzzy numbers between existing methods and proposed method is presented as in below table 1.

Table 1. Comparative Analysis of Different Ranking Methods

In the below comparison table 2, we can observe that the ranking techniques having their won importance and drawbacks. We can straightly state that each technique can apply for a particular case in real life.

Table 2. Comparative Analysis of Ordering in Different Ranking Methods.

5. Results and Discussion

Several numerical examples were used to test the suggested ranking procedure for Trapezoidal Intuitionistic Fuzzy Numbers (TrFNs) based on the orthocenter of centroids. The outcomes are contrasted with those of other ranking techniques, such as the centroid-based ranking technique, the area-based ranking method, and the score-based ranking system .The results presented in Table 1 &2 demonstrate that the proposed ranking process based Euclidean distance on the orthocenter of centroids outperforms existing methods in terms of accuracy. The proposed method provides a more reliable and robust ranking of TIFNs, which is essential in decision-making applications. The method is time consuming and simple to calculate compared to other existing methods. This ranking technique can apply for both TIFNs and TrIFNs which refers the uncertainty. It provides accurate results comparative to other methods because it measures Euclidean distances between the two referring points which are derived from orthocenters of centroids of membership and non-membership functions. Using this ranking technique several transportation problems, optimization problems and decision making problems of vague nature can be solved.

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