Characterization of the Interference Distribution for Large-scale Wireless Communication Network

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Abstract

The research paper discusses the characterization of aggregate interference distribution for large-scale wireless communication networks. The paper proposes an innovative approach to estimate the probability density function of the aggregate interference in the wireless network by using statistical modeling techniques. The methodology involves using a DS-CDMA communication system that captures aggregate interference in large-scale wireless networks, considering factors like path loss, shadowing, fading, and spatial distribution of interfering nodes. It was unearthed that the interference of the large-scale wireless network was alpha-stable probability density function (pdf) of Weibull distribution, which belongs to the class of heavy-tailed probability distributions. Experimental results from simulations on large-scale wireless communication networks using Matlab 20021a were used to validate the effectiveness and accuracy of the proposed approach, demonstrating the reliability of the statistical modeling technique. The alpha-stable pdf model assist network designers and engineers in gaining valuable insights into aggregate interference characterization and to make informed decisions to optimize network performance.

Keywords: *Large-scale Wireless Communication Network, Aggregate Interference, Probability Density Function, Alpha-stable Probability Distribution*

1.0 Introduction

The characterization of interference distribution in large-scale wireless communication networks is crucial for understanding and optimizing performance. Interference refers to unwanted signals that degrade the quality of the desired signal. As networks become denser and more complex, accurately characterizing interference distribution becomes more challenging. Factors such as network topology, channel conditions, transmission power levels, and other interfering devices influence interference. Mathematical modeling and empirical measurements are two common approaches for analysing and predicting interference patterns. Stochastic geometry is a commonly used mathematical model for analysing random spatial patterns and their statistical properties. Empirical measurements collect real-world data from operational wireless networks to understand interference patterns and statistics. Accurately characterizing interference is essential for designing efficient resource allocation strategies, interference mitigation techniques, and adaptive transmission schemes to enhance service quality.

1.1. **Problem statement**

Suppose $X_1, X_2, ..., X_N$ are observed independent and identical realization of a symmetric alpha stable ($S\alpha S$)random variables. Define

$$Z_n = \max(X_1, X_2, ..., X_n)$$

$$W_n = \min(X_1, X_2, ..., X_n)$$

and

$$F(x) = P(X_i < x)$$

where $P(Z_n < x) = H_n = F^n(x)$ and $P(W_n \geq x) = 1 - L_n(x) = (1 - F(x))^n$

of unknown characteristic exponent α , skewness parameter β , location parameter δ , and dispersion γ . We seek to estimate the exact parameter values of the $S\alpha S$ distribution of X_i from the observed realization in a metropolitan wireless communication interference.

1.2. **Objective of the study**

The study aims to identify stable probability density functions for describing aggregate interference in large-scale wireless communication networks and understand its impact on network performance.

1.3. **Significance of the study**

The study significantly impacts wireless communication performance, reliability, resource allocation algorithms, and power control strategies, enhancing network capacity, minimizing interference, and improving energy efficiency.

2.0 **Literature Review**

In statistics, to fully characterize a distribution means to state central tendency, dispersion, and its shape. Over here, the moment-generating function plays a major role. This describes all its essential properties and features of the distribution.

2.1 **Aggregate Interference**

Interference is the obstruction of signals from the transmitter to the receiver, with the quantum of interference determining the difference between transmitted and received signals, and aggregate interference summarizing all interference signals due to transmission, distortion, and propagation. The major components of interference are external, co-channel, adjacent channel, intermodulation, and noise.

2.2. Theoretical Framework

The theoretical framework model is based on the theoretical links between normalizing constants, asymptotic distribution, and weak convergence [1]. From the theory, the existence of normalizing constants ensures the existence of asymptotic distribution. Moreover, these normalizing constants also make the extrema have limited distribution and nondegenerate distribution functions. The limit distribution becomes a sufficient condition for the existence of weak convergence and that necessary condition for weak convergence is the existence of a nondegenerate distribution function [1]. An estimator of a given parameter is said to be consistent if it converges to the true value of the population parameter as the sample size increases. This information is illustrated in Figure 1 below.

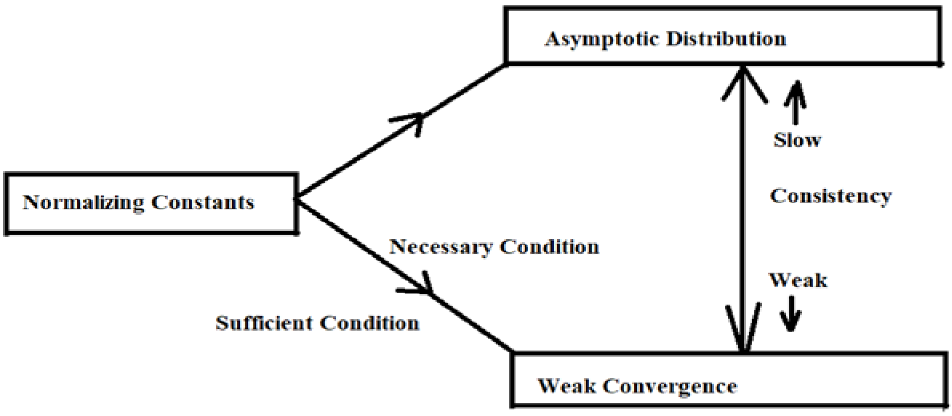


Figure 1: Theoretical Framework

Moreover, consistency is the factor that links weak convergence theory to asymptotic distribution. Whenever the consistency is weak, convergence is weak and when consistency is slow, we say the distribution is asymptotic. Also, any consistent sequence of random variables in maximum likelihood estimation is asymptotic normal and asymptotic efficient. Whenever the consistency of the signal in the medium is both slow and weak, we have an asymptotic -weak -convergence theorem [2-4].

2.3. Alpha-Stable Distribution

Stable distributions are the only ones that satisfy the generalized central limit theorem of the distribution; their statistical properties are determined by the four parameters of the characteristic functions, which are the characteristic index (exponent) α, symmetric coefficient β, coefficient of dispersion $\gamma(\gamma > 0)$, and position parameters δ. For characteristic index, $\alpha \in [0,2]$. The Gaussian (Normal) distribution is a subset of the stable distribution where $\alpha = 2$. The alpha-stable distribution has no analytically unified explicit probability density function. The characteristic function is typically used to characterize it [5]. The pdf is known for $\alpha = 0.5, 1,$ and $2,$ but when α takes other values apart from the mentioned values, the pdf is unknown. According to [6], the pdf of alpha-stable distribution is defined as

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(t) e^{-jxt} dt \text{.................K K ....1}$$

where

$$\varphi(t;\alpha,\beta,\gamma,\delta) = \begin{cases} \exp\left(jt\delta - |\gamma t|^{\alpha} - j\beta \operatorname{sgn}(t) \tan\left(\frac{\pi\alpha}{2}\right)\right), & \alpha \neq 1 \\ \exp\left(jt\delta - |\gamma t|^{\alpha} - j\beta \operatorname{sgn}(t) \frac{2}{\pi} In(t)\right), & \alpha = 1 \end{cases} \text{........2}$$

where

$$sign(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

The existence of the moments of the alpha-stable distribution depends only on the characteristic index of the alpha stability of the random variable.

$$\begin{cases} E\left(|x|^{p}\right) < \infty, & p \in]0,\alpha[\text{.............}3a \\ E\left(|x|^{p}\right) = \infty, & p \geq \alpha \text{.................}3b \end{cases}$$

Equation 3a is known as a Fractional lower-order moment (FLOM) where the moment exists and equation 3b does not exist [7].

2.3.1. **Properties of Alpha Stable Distribution**

1. They exhibit characteristics of independent and identical distributed random variables. This means that at their extremes are asymptotic and weak convergence properties.

2. There exists a weak limit of the distribution function of F_n as $n \to \infty$. That is

   $F_n \to G \quad n \to \infty.$

3. They have four parameters, which define their characteristic functions.
4. They have regular varying properties. That is $X \sim S_\alpha(\sigma, \beta, \mu) \quad with \quad \alpha \in]0,2]$
5. They have lower-order moments when the order is less than two.
6. It is an analytic function [6].

3.0 Methodology

3.1 Experimental Setup

The block diagram below is the DS-CDMA communication scheme, based on chaotic dynamic is the equipment that was used to gather the data in the wireless communication. The system is composed of three sections namely; signal generator and recorder, transmitter, synchronization, and receiver sections.

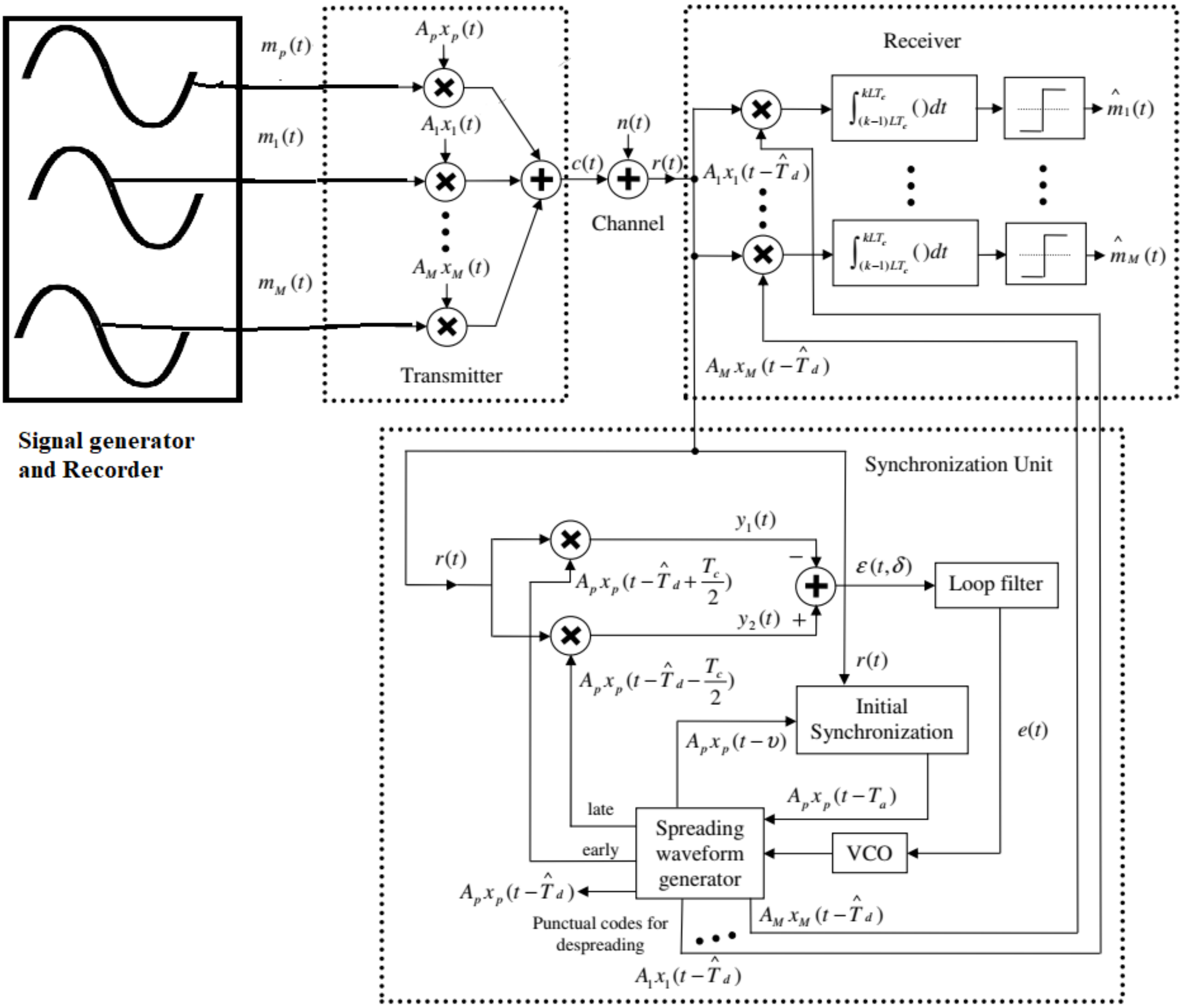


Figure 2: DS-CDMA chaotic communication system with synchronization unit

A signal is generated, recorded, and transmitted to the receiver. Transmitted additive white Gaussian noise is added to the channel. At the receiver, the transmitted signal is synchronised, received, and recorded. The difference between the transmitted signal and the received signal is the attenuated signal due to interference which will be called aggregate interference.

3.2 Extreme Value Theory

Extreme value theory (EVT) focuses on large departures from the median of probability distributions. It aims to determine the likelihood of events that are more extreme than any previously recorded events from a given ordered sample of a given random variable. Extreme value analysis is frequently employed in a variety of fields, including geological engineering, finance, earth sciences, the forecast of traffic, and communication [1].

EVT uses the extrapolation of very small probabilities that are involved in the evaluation of the effectiveness of communication devices and signal processing algorithms. In addition, EVT is used in the estimation of the characteristic exponent of the model for impulsive interference. This is particularly true in the fields of communication theory and signal processing. However, the area of EVT has remained largely unpopularized in the communications and signal processing sectors [1].

3.3 Extreme Value Theory of Weibull Distribution

According to [19] the pdf of the Weibull distribution, which belongs to the family of extreme value distributions is defined as

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} \exp\left(-\left(\frac{x-a}{b}\right)^{c}\right) \text{.......................} 4$$

where $x \geq a,\ a \in \mathbb{R}$, $b, c \in \mathbb{R}^{+}$ with the cumulative distribution function (cdf) given as

$$F(x) = \int_{0}^{x} f(u)du = 1 - \exp\left(-\left(\frac{x-c}{b}\right)^{c}\right) \text{..................} 5$$

with the hazard function (HR) defined as

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c} \text{........................} 6$$

$a, b, and\ \ c$ are constants.

These distributions are the limit distributions of the smallest or the greatest value respectively in a sample with a sample size $n \to \infty$.

For a finite sample of size *n* with each sample variable $X_i\left(i \in [1,n]\right)$ being independently and identically distributed with pdf $f(x)$ and cdf $(F(x))$ define on two statistics

$$Y_n := \min_{i \in [1,n]}\{X_i\} \text{..................................} 7a$$

and

$$Z_n := \max_{i \in [1,n]}\{X_i\} \text{..................................} 7b$$

The Y_n and Z_n distributions are respectively sample minimum and sample maximum, and are defined as follows:

$$\Pr\left(Y_n > y\right) = \Pr\left(X_i > y \quad \forall i\right)$$

$$= \Pr\left(X_1 > y, X_2 > y, \ldots, X_n > y\right)$$

$$= \left(1 - F(y)\right)^n$$

Hence, the cdf is

$$F_{Y_n}(y) := \Pr\left(Y_n \le y\right) = 1 - \left(1 - F(y)\right)^n \text{..............................} 28a$$

With the pdf

$$f_{Y_n}(y) := nf(y)\left(1 - F(y)\right)^{n-1} \text{..} 28b$$

Moreover,

$$\Pr\left(Z_n \le z\right) = \Pr\left(X_i \le z \quad \forall i\right)$$

$$= \Pr\left(X_1 \le z, X_2 \le z, \ldots, X_n \le z\right)$$

$$= \left(F(z)\right)^n$$

Hence, the cdf is

$$F_{Z_n}(z) := \Pr\left(X_n \le z\right) = \left(F(z)\right)^n \text{.................................} 29a$$

With corresponding pdf

$$f_{Z_n}(z) := nf(z)\left(F(z)\right)^{n-1} \text{...} 29b$$

Both Y_n and Z_n limiting distributions as $n \to \infty$ is degenerated. At this point it is critical to unearth existing non-trivial limiting extreme value distribution and mathematically view its functional characteristics, which will characterize aggregate interference.

According to [8], R.A Fisher and L.H.C. Trippet were the first to derive the asymptotic behaviour of extreme value distribution, which belong to the class of heavy-tailed distribution. The properties of extreme value distributions are as follows: they are analytic in nature, have asymptotic behaviour, limiting distribution, and weak convergence.

Again, whenever the probability of an observation is less than x, the probability that the greatest of a sample of size n is less than x is P^n. It is assumed that $P^n(x)$ partly varies as $a_n x$ and b_n. Hence, we have

$$P^n(x) = P(a_n x + b_n)$$

which is a functional equation from the limiting distribution.

Solving this analytic function using functional analysis, the only possible existing limiting curves are

I. $dP = e^{x-e^{-x}} dx................................8a$

II. $dP = kx^{-(k+1)} e^{x^k} dx..........................8b$

III. $dP = k(-x)^{(k-1)} e^{-(-x)^k} dx..........................8c$

which is exactly the order of enumeration. This result later gave birth to three classes of extreme value distribution (for maximum) of type I, type II, and type III.

Due to

$$\min_i (X_i) = -\left(\max_i (-X_i)\right)..............................9$$

the distribution of the asymptotically extreme sample values corresponds as follows where Y and Z are respectively minimum and maximum variables:

$$\Pr(Y \le x) = \Pr(Z \ge -x) = 1 - \Pr(Z \le x)....................10a$$

$$F_Y(x) = 1 - F_Z(-x)..................................10b$$

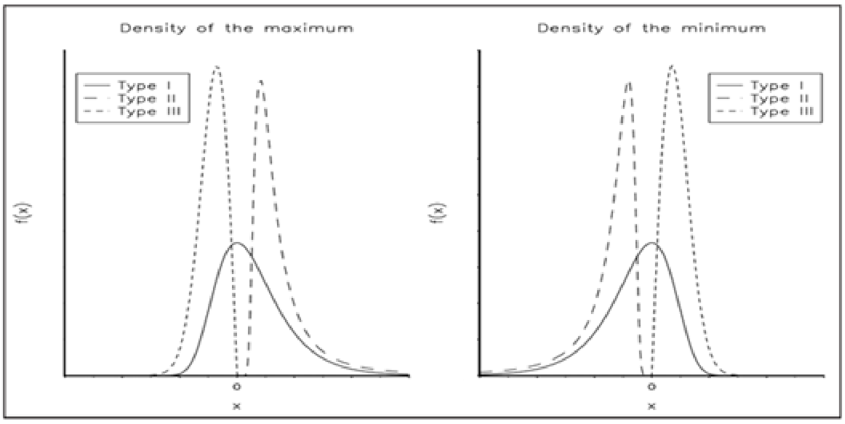
$$f_Y(x) = 1 - f_Z(-x)...................................10c$$

The comparison of the extreme value densities is illustrated in the table 1 below.

Table 1: Density Functions of Extreme value Weibull distribution

	Maximum	**Minimum**
Type I	$f(x) = \exp(-x - e^{-x}), \quad x \in$	$f(x) = \exp(x - e^{-x}), \quad x \in$
Type II	$f(x) = \frac{k}{x^{k+1}} \exp(-x^{-k}), \quad x \ge 0$	$f(x) = \frac{k}{(-x)^{k+1}} \exp(-(-x)^{-k}), \quad x \le 0$
Type III	$f(x) = k(-x)^{k-1} \exp(-(-x)^k), \quad x \le 0$	$f(x) = k(x)^{k-1} \exp(-x^k), \quad x \ge 0$

From Table 1, X_i is the continuous. The empirical graph of the maximum probability density function is the reflection of the type-equivalent minimum probability density function, reflection about the ordinate axis is illustrated in the figure 3 below.



A **B**

Figure 3: Empirical Graph of the pdf

3.4. Generalized Extreme Value of Weibull Distribution

The cdf of GEVD is given by

$$F_X(x) = \begin{cases} e^{-(1+\xi((x-\mu)/\sigma)^{-1/\xi}}, & x \in \left]-\infty, \mu - \sigma/\xi\right] \quad for \quad \xi < 0 \\ e^{-(1+\xi((x-\mu)/\sigma)^{-1/\xi}}, & x \in \left[\mu - \sigma/\xi, 0\right[\quad for \quad \xi > 0 \\ e^{-e^{-(x-\mu)/\sigma}}, & x \in \left]-\infty, \infty\right[\quad \xi = 0. \end{cases} \quad \ldots\ldots\ldots\ldots\ldots 11$$

When equation 11 is differentiated it yields equation 12 which is the pdf

$$f_X(x) = \begin{cases} \frac{1}{\sigma}\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right) \times e^{-(1+\xi((x-\mu)/\sigma)^{-1/\xi}}, & x \in \left]-\infty, \mu - \sigma/\xi\right] \quad for \quad \xi < 0 \\ \frac{1}{\sigma}\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right) \times e^{-(1+\xi((x-\mu)/\sigma)^{-1/\xi}}, & x \in \left[\mu - \sigma/\xi, 0\right[\quad for \quad \xi > 0 \\ \frac{1}{\sigma} e^{-(x-\mu)/\sigma} \times \quad e^{-e^{-(x-\mu)/\sigma}}, & x \in \left]-\infty, \infty\right[\quad \xi = 0. \end{cases} \quad \ldots\ldots\ldots\ldots \; 12$$

[19].

4. Data Analysis

In this section, the results of the experimental work together with the derived theoretical expressions for characterization of the interference of metropolitan wireless communication are presented. Figure 5 below shows the various probability distributions that were fitted with Matlab.

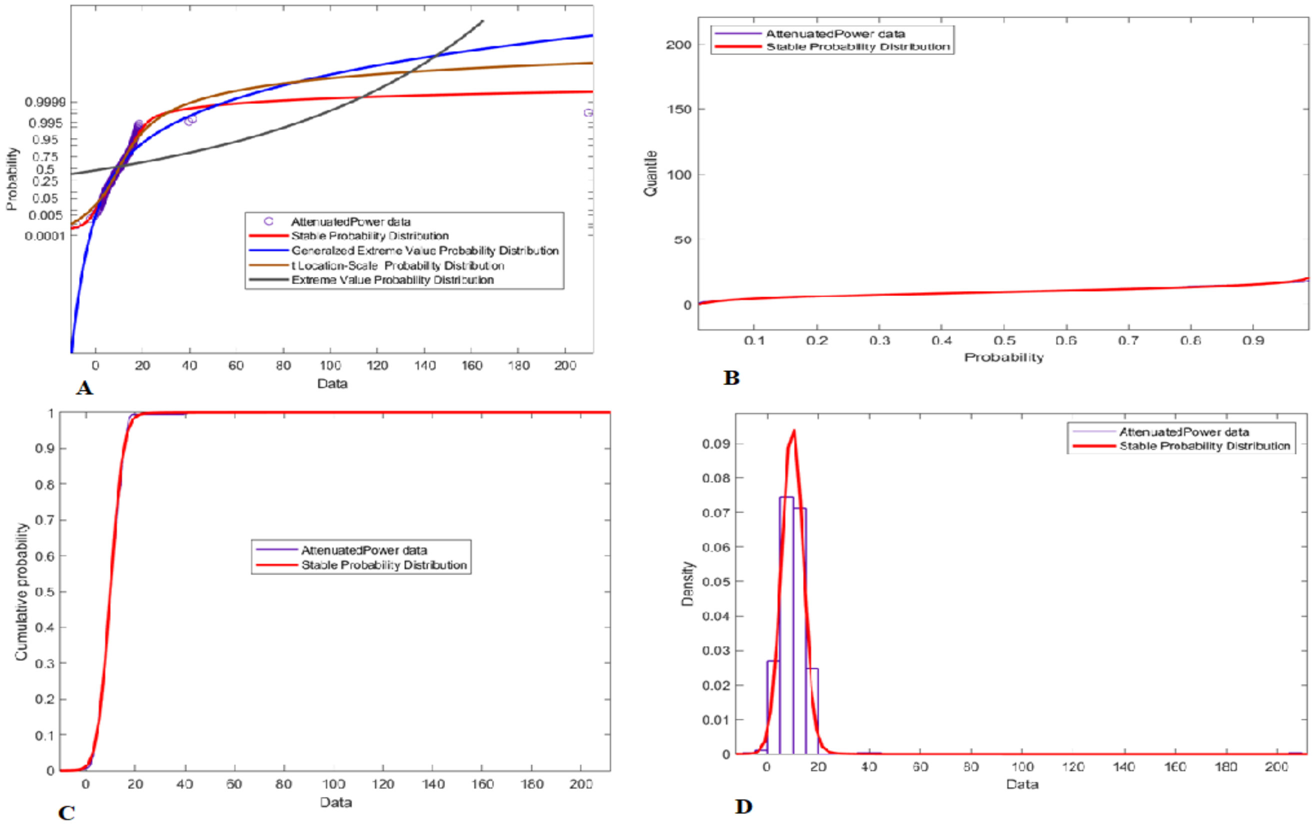


Figure 4: Graph of experimental data

Figure 4 A, B, C, and D, are respectively the probability plot, quantile, cdf, and probability mass function with stable probability distribution of the experimental data. Figure 4A shows the various probability functions tested to fit the data. Figure 4B is a quantile plot of the alpha-stable pdf whereas Figure 5C is the cdf of the data. Figure D shows a histogram of the data obtained from the experiment with an imposition of a stable probability distribution.

From the probability plot (i.e. Figure 3A), the best-fitted probability distribution function at the base stations within the metropolitan area is alpha -stable probability distribution function at a 95% confidence interval with the following statistics presented in Table 2.

Table 2: Statistics of the distribution at the transmission base station

Distributio n	**Alpha Stable**

Log likelihood	-1618.1			
Domain	$y \in]-\infty, \infty[$			
Mean	9.85825			
Variance	NaN			
Parameter	Estimation	Standard Error		
Alpha	1.94865	0.0374347		
Beta	0.384452	0.585758		
Gam	2.96007	0.0990811		
Delta	9.76625	0.203176		
Estimated Covariance				
	Alpha	Beta	Gam	Delta
Alpha	1.40135×10^{-3}	3.7324×10^{-3}	1.10446×10^{-3}	9.89512×10^{-4}
Beta	3.7324×10^{-3}	3.43112×10^{-1}	4.98697×10^{-3}	-4.90619×10^{-2}
Gam	1.10446×10^{-3}	4.98697×10^{-3}	9.81709×10^{-3}	9.26215×10^{-4}
Delta	9.89512×10^{-4}	-4.90619×10^{-2}	9.26215×10^{-4}	4.12804×10^{-2}

From probability theory and mathematical statistics of maximum log-likelihood estimation combined with graphical theory, the aggregate attenuated signals due to interference is Alpha-stable distributed at. From Table 2 presents a 95% confidence interval estimation for the parameters of the alpha-stable distribution to be $\alpha = 1.95, \beta = 0.38, \gamma = 2.96, and\ \delta = 9.77$ at 2 decimal places. The characteristic function of Alpha-stable distribution is given as

$$\varphi(t;\alpha = 1.95, \beta = 0.38, \gamma = 2.96, \delta = 9.77) = \exp\left(9.77jt - 2.96|t|^{1.95} - 0.38j\,\mathrm{sgn}(t)\tan\left(\frac{1.95\pi}{2}\right)\right) \ldots\ldots \quad 13$$

$$sign(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

Replacing jt by θ, we have the MGF of stable distribution as

$$M(\theta) = \exp\left(9.77\theta - 2.96|-j\theta|^{1.95} - 0.38\operatorname{sgn}(\theta)\tan\left(\frac{1.95\pi}{2}\right)\right) \ldots\ldots\ldots\ldots \quad 14$$

where

$$sign(\theta) = \begin{cases} -1, & \theta < 0 \\ 0, & \theta = 0 \\ 1, & \theta > 0 \end{cases}$$

at 95% confidence interval.

This Alpha-stable distribution is non-Gaussian in nature, non-symmetrical, and skewed to the right. Since α-stable random variables do not in general have analytical closed-form probability density functions, the characteristic function or moment generating function plays a key (special) role.

Another important factor is the standardization of alpha stable distribution. A stable distribution is said to be standardized, if and only if dispersion, $\gamma = 1$ and location parameter, $\delta = 0$. Since Alpha- stable distribution is the only non-Gaussian distribution that obeys stability property and generalized central limit theorem. It is important to deduce standard $S\alpha S$ density function for the characteristic exponent. A Standardized random variable X of a stable distribution with four parameters is given as

$$z = \frac{X-\delta}{\gamma^{1/\alpha}} = \frac{X-9.77}{2.96^{1/1.95}} = \frac{X-9.77}{1.745}$$

This standardization transforms the distribution into a standardized $S\alpha S$ probability density function.

The non-existence of a pdf of alpha-stable distribution is possible for its existence at fractional lower order moment. Using the power series probability density function, $S\alpha S$ random variable with zero-centeredness and a unity dispersion can be characterized. This was derived by Russian Mathematician which was later translated to English. This is given as

$$_{3}f_{\alpha}(z) = \begin{cases} \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\boldsymbol{n!}}\Gamma(\alpha n+1)\mathrm{Sin}\left(\frac{n\alpha\pi}{2}\right)|z|^{-\alpha n-1}, & \alpha \in]0,1[, z \neq 0 \\ \frac{1}{\pi(z^2+1)}, & \alpha = 1 \\ \frac{1}{\pi\alpha}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}\Gamma\left(\frac{2n+1}{\alpha}\right)z^{2n}, & \alpha \in]1,2[\\ \frac{1}{2\sqrt{\pi}}e^{-z^2/4}, & \alpha = 2 \end{cases}$$

But $\alpha = 1.95 \in]1,2[$, therefore

$$f_3(z) = \frac{1}{1.95\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}\Gamma\left(\frac{2n+1}{1.95}\right)z^{2n} = \frac{1}{1.95\pi}\sum_{n=0}^{\infty}(-1)^n\left(\frac{\Gamma\left(2n+1/1.95\right)}{\Gamma(2n+1)}\right)z^{2n},$$

Analytically,

$$\lim_{n\to\infty}\left(\frac{\Gamma(2n+1/1.95)}{\Gamma(2n+1)}\right) = \frac{\infty}{\infty}\left(In\det er\min ate\right)$$

This was because the gamma function is a factorial function that is monotonic non-decreasing making the coefficient of the power series probability density function graph inexpressible. Moreover, mathematically observing the function critically, the alpha-stable function behaves like a Gaussian density function due to the term x^{2n} which makes the function bell-shaped or quadratic shape. The gamma function generates the coefficient of the power series. Moreover, the gamma function is a factorial function that is monotonic non-decreasing to infinity. As $n \to \infty, gamm(n) \to \infty$, $f_3 \to \infty$. This makes the pdf of the stable distribution non-existent. Hence the non-existence of the stable distribution function problem has not been solved and needs to be solved. The researchers are mathematically confident of the existence of the probability density function which can be derived or modelled mathematical to yield the alpha-stable distribution generated by the data using numerical method. This testifies to the mathematical fact that whenever the analytical method fails, the only option available to use is numerical.

Derivation of Existing Alpha -Stable Probability Density function with $\alpha = 1.95$

According [19] a modified Weibull distribution is given by

$$f_3(x) = \exp\left(\frac{x-\mu_\omega}{a\alpha} - \frac{1}{b\alpha}\exp\left(\frac{x-\mu_\omega}{c\alpha}\right)\right)\Bigg|_{\substack{\alpha=1.95\\ \mu_\omega=9.85825}} = \exp\left(\frac{x-9.85825}{1.95a} - \frac{1}{1.95b}\exp\left(\frac{x-9.85825}{1.95c}\right)\right)\text{.............} 15$$

where $a, b,$ and c are constant, μ_w is the mean and α is the standard alpha stability parameter for stable probability distribution functions.

Fitting the data obtained, we used the following parameter values

$a = 1.35, b = 0.9, c = 2, \mu_w = 9.85825$ and $\alpha = 1.95$ to obtain the Figure below.

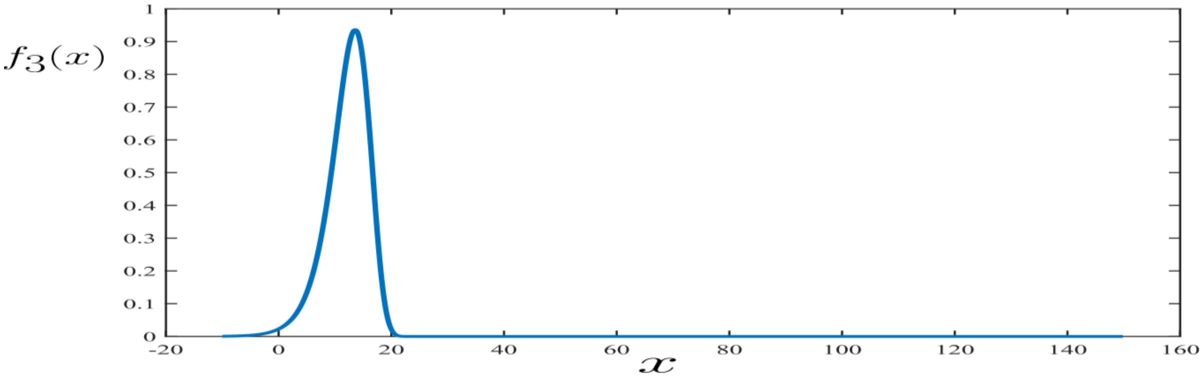


Figure 5 : Derived PDF of Stable Distribution Function with $\alpha = 1.95$

Therefore, the pdf of the aggregate interference is

$$f_3(x) = \exp\left(\frac{x-9.85825}{2.6325} - \frac{1}{1.755}\exp\left(\frac{x-9.85825}{3.9}\right)\right) \quad \text{............} 16$$

Comparing Figures 4D and 5 graphically, figure 5 has the same characteristics as Figure 4D. Both behave like the Gaussian at the origin but lose it Gaussianity when it departs from the origin. Another critical feature of the behaviour of the function observe from the graph is the signal is asymptotic to the horizontal axis as time elapses $f(x) \to 0, \quad as \quad x \to \infty.$ This behaviour goes along with weak convergence of the signals of the interference as it has been mathematically proven in the previous pages. Hence characterization of large-scale wireless communication aggregate interference depends on asymptotic weak convergence theorem. Theoretically, it implies the existence of limit distribution functions and normalizing constants. It is this theorem that this paper depends on and becomes a contribution to the body of researchers and academia.

A predictive implication of this is the researchers are 95% confident that optimal detection in large-scale wireless communication will be asymptotic since every aspect of data transmission and reception is affected by interference.

Conclusion

We conclude that in the characterization of the interference distribution for a large-scale wireless communication network, the aggregate interference exhibits the behaviour of alpha-stable distribution $\alpha = 1.95, \beta = 0.38, \gamma = 2.96, and\ \delta = 9.77$ which is unimodal for all $\alpha \in]1, 2]$, analytic function, asymptotic and weak convergence, and the pdf of the aggregate interference is a Weibull distribution. A value close to 2 (around 1.95) indicates significant variability in the distribution, potentially impacting system performance. This high alpha-stable value implies packet collisions, reduced signal quality, and decreased network throughput. Understanding this is crucial for optimizing network design and operation.

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