Fuzzy Soft Sets in Collaborative Decision-Making: Bridging Uncertainty and Consensus

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Abstract: A useful technique for dealing with ambiguity and impreciseness in judgment situations is soft set theory. To assess how well soft sets, fuzzy soft sets, and fuzzy soft sets handle ambiguity and variability, this study investigates their use in a group decision-making framework. Fuzzy soft sets expand the adaptable framework of soft sets by adding degrees of membership, allowing for a more thorough examination of challenging decision-making issues. These techniques are especially useful when making decisions in groups since they make it easier to incorporate different personal preferences, which enhances the process of reaching consensus. Despite their benefits, soft sets and fuzzy soft sets have not yet reached their full potential in group decision-making, particularly when resolving opposing preferences and differing degrees of certainty. This study's main goal is to compare these methods' accuracy, dependability, and effectiveness. Through their application to a real-world example involving the selection of the most suitable clinic for physiotherapy from a pool based on a number of criteria and the preferences of a panel of decision-makers, the study shows how all three approaches can effectively manage uncertainty and improve decision-making outcomes.

Keywords: Soft set, fuzzy soft set, decision-miakng, uncertainty, ambiguity, physiotherapy.

1. Introduction

Conventional approaches frequently fail to yield trustworthy results in decision-making processes where imprecision and uncertainty predominate. A strong framework for addressing these issues is provided by soft set theory, which was created to deal with ambiguous data and makes decisions in unpredictable situations easier. Fuzzy soft sets that incorporate degrees of membership improve soft sets' descriptive capabilities and increase their adaptability for complex decision-making scenarios that call for detailed criteria. This adaptability comes in quite handy when dealing with ambiguous or subjective data.

Making decisions as a group adds another level of complexity since it necessitates combining different preferences and points of view, which frequently results in competing standards that make consensus difficult. These issues are addressed by fuzzy soft sets in group decision-making frameworks, which provide for an organized method of reaching consensus by methodically quantifying and aggregating individual preferences. Fuzzy soft sets are a dependable method for resolving group problems in uncertain situations because they effectively balance divergent viewpoints and enhance the process of making decisions.

By means of the same problem, this study investigates the efficacy of three different approaches: fuzzy soft sets, soft sets, and fuzzy soft sets in a group decision-making setting. Assessing their precision, reliability, and effectiveness is the aim. This study intends to demonstrate the potential of fuzzy soft set theory as an effective paradigm for enhancing choice quality in uncertain situations, particularly in group scenarios, by examining how each approach handles ambiguity and aids in decision-making.

The document is set up as follows to help arrange the discussion: Research gaps are identified and the pertinent literature is reviewed in Section 2 • The basic ideas and characteristics of soft sets and fuzzy soft sets are covered in Section 3. Useful examples showing how these techniques support actual decision-making situations are given in Section 4. The conclusions and results obtained from the examples are shown in Section 5. Lastly, Section 6 highlights the study's distinctiveness and summarizes its contributions.

2. Literature Review

A significant development in mathematical techniques for handling uncertainty in a variety of domains is soft set theory. Because of its adaptability, it can successfully convey complicated and confusing information, particularly as uncertainty grows in importance. Molodtsov[1] introduced the concept of soft sets, offers a more reliable foundation than conventional techniques, which frequently have trouble with uncertainty in the actual world. To solve decision-making problems in imprecise environments, Roy and Maji [2] presented a unique method for object detection using ambiguous multi-observer data. Their approach involved employing a fuzzy soft set to create a comparison table in a parametric environment to help with decision-making. Maji et al. [3] used soft set theory and rough mathematics to address a challenge involving decision-making. Zadeh [4] was the first to define a fuzzy set as a collection of elements with varying degrees of membership. Each element's membership is determined via a characteristic the process that determines a involvement score, ranging from zero to one. A more comprehensive understanding of fuzzy soft set decision-making is provided by Feng et al. [5]. They stressed that the choice value approach was designed for scenarios that were clear-cut and cannot effectively handle fuzzy soft set decision-making challenges. By combining soft set and multi-fuzzy soft set models, Yang et al. [6] introduced the concept of multi-fuzzy soft sets and provided examples of how to apply this tactic in decision-making scenarios. Çağman et al. [7] presented the notion of intuitionistic fuzzy soft sets for decision-making and shown its effectiveness for a range of uncertain real-world problems. In their theoretical study of soft sets, Husain et al. [8] focused on De Morgan's rules, providing proofs and looking into other universal laws related to the framework. According to Sooraj et al. [9], the inherent uncertainty and knowledge gaps sometimes make it impossible for a single decision-maker to make informed decisions in real-life scenarios. Tripathy et al. [10] pointed out the flaws in the methods used now and offered solutions to increase the efficacy and realism of the decision-making process. To address decision-making problems, Jana et al. [11] combined bipolar intuitionistic fuzzy soft sets with soft sets. Khalil et al. [12] introduced the concept of inverse fuzzy soft sets in their 2019 study, outlining its characteristics, properties, and operations. They also used two decision-making scenarios to demonstrate the applicability of this approach. , Bipolar fuzzy soft mappings (BFS-mappings) and bipolar fuzzy soft sets (BFS-sets) were employed by Riaz et al. [13] to control bipolarity in medical diagnostics and create a reliable mathematical model for accurate diagnosis and therapy recommendations. Begam et al. [14] constructed a similarity measure for lattice-ordered multi-fuzzy soft sets using a set-theoretic technique and applied it to decision-making. A novel soft set theory for making decisions in the presence of uncertainty is presented by Dalkılıç [15]. Zulqarnain et al. [16] demonstrated the application of their suggested TOPSIS method, which is based on correlation coefficients, in decision-making. In addition to presenting decision-making strategies based on fuzzy soft competition hyper graphs, Akram et al. [17] developed a novel framework that demonstrated the connection between fuzzy soft sets and hyper graphs. In a fuzzy environment with trapezoidal interval type-2, Chen et al. [18] coupled DEA (data envelope analysis) and BWM (best-worst method) using a fuzzy group decision-making technique with many criteria. While releasing a similarity formulation and comparing it with other models, Rahman et al. [19] talked about the universality of their proposed structure. By illustrating how to use TOPSIS approach founded on correlation coefficients to decision-making, Salsabeela et al. [20] demonstrate its application. Taköprü et al. [21] state that when looking at the components that make up soft elements, the concepts of a soft element and a soft connection can be useful and appropriate. These elements are equivalent to single-element soft sets, and a soft element provides a pattern that illustrates the links between the alternatives and which alternative from the soft set is preferable for each descriptive characteristic. like graph energy's function in graph theory. Mudrić et al. [22] incorporate a feature from graph theory into the theory of fuzzy soft sets, which conceptually deviates from graph theory, and provide new parameters that describe the nature of fuzzy soft sets. This contributes to the advancement of fuzzy soft set theory application. Soft set theory was utilized by Orhan et al. [23] to suggest two methods. By examining the connections between the symptoms, the first algorithm calculates the possible influence of each symptom on the others. Finding the most prevalent symptom is the second step. The outcomes of using both algorithms show that, in order to effectively control the pandemic, it is more beneficial to look at diverse places.

2.1. Research Gap

Despite notable progress, there are still large gaps in the use of fuzzy soft sets and soft sets for decision-making, especially in ambiguous group contexts. The complexity of group dynamics is frequently overlooked in favor of individual decision-making in the majority of the research that is currently available. Although sophisticated techniques like intuitionistic fuzzy, multi-fuzzy, and bipolar fuzzy soft sets have been put forth, nothing is known about how to incorporate them into a cohesive framework for collective decision-making. There are very few comparative studies of methods such as inverse fuzzy soft sets and fuzzy hypergraphs. Moreover, there is little attention paid to real-world applications, such as computational and graphical models.

By investigating and contrasting the use of fuzzy soft settings and soft sets in a group decision-making framework, our study seeks to close these gaps. The study demonstrates how these approaches might improve decision-making outcomes in collaborative and uncertain situations by evaluating their accuracy, effectiveness, and capacity to handle ambiguity in a standardized environment.

3. Preliminaries

Soft set: Assume that U represent the original universal set and E denote the collection of all potential parameters related to U. Parameters typically correspond to features, traits, or qualities of the items in U. The technical definition of a soft set over U is as follows:

If F maps E to the power set of U, represented by the symbol P(U), then the pair (F,E) is called a soft set over U. Specifically, $F:E\rightarrow P(U)$, where P(U) is the sum of all subsets of U.

To put it simply, a soft set is a parameterized collection of subsets of U. The e-elements of the soft set (F,E) or the e-approximate elements inside the soft set are represented by the set F(e) for each $e\in E$.

Fuzzy Soft set: Consider a universal set U and a collection of parameters E. Let I^U represent each and every fuzzy subset of U.

Suppose A is a subset of E. A fuzzy soft set over U is a pair (F, E) where F is a function that maps each parameter in A to a fuzzy subset of U.

This mapping is written as: $F: A\rightarrow I^U$

F is a function that connects each characteristic in A to a fuzzy subset of U. In other words, F tells us, for each characteristic, how well each item in U matches that characteristic. So, F takes a characteristic from A and gives you a fuzzy subset of U.

Membership Function for Fuzzy Soft Sets:

Here is how we define the membership function for any a∈Aa:

$$\mu^{a}_{(F,A)}(X) = \alpha, \alpha \in [0,1] \qquad (1)$$

This means that for each parameter a in the set A, the membership function $\mu^{a}_{(F,A)}(X)$assigns a value between 0 and 1, representing the degree of membership.

4. Methods

4.1. Soft Set Theory in Decision Making

Think about a situation where Dr. A, a physiotherapist, wants to suggest the finest physiotherapy clinic for patients with nerve-related problems based on particular physiotherapy-related criteria. He assesses every clinic according to a set of standards that define the "best clinic" for the patient. The parameters are E= {specialized equipment, certified physiotherapists, hygiene, patient happiness, location, nerve rehabilitation programs, post-treatment support}. Let {CL1, CL2, CL3, CL4, CL5, CL6} stand for six clinics. The following future algorithm uses soft set theory to tackle the aforementioned difficulty.

Algorithm: Optimal Element Selection Based on Parameter Matching
Inputs:

1. The universal set U
2. The entire parameter set E
3. A subset of the parameters used to make decisions P

Output:

- The component of U that best meets the requirements specified by P.

Step 1: Set up the tracking structures.

- Construct the match_tracker dictionary, in which each value monitors the parameter match count (starting at 0) and each key denotes an element u∈U.
- Create optimal_element and set its initial value to None.
- Set the value of max_matches to 0.

Step 2: Assess Each Element's Parameter Matches

- For every u in U:
    1. Find the number of parameters in P that u satisfies.
    2. Use this count to update the matching value in match_tracker.
    3. If the match count is greater than max_matches:
        - Update max_matches with the new number.
        - Set optimal_element to u for the time being.

Step 3: Determine the Top Matching Element(s)

- Take out every element in match_tracker with a match count of max_matches. Keep them in the top_contender list.
- If there is just one element in top_ contender, assign it to optimal_element.

Step 4: If required, secondary selection

- If top_ contender contains more than one element:
    - Use a secondary criterion, such as a tie-breaking rule, an extra property, or priority ranking.
    - Select the element as optimal_element that satisfies this requirement.

Step 5: Verify Optimality and Return

- If the number of parameters in P is equal to max_matches, then optimal_element is the best match.
- Give back optimal_element as the outcome

Solution of the above problem by using the Algorithm

In order to recommend the best physiotherapy clinic in U for patients with nerve-related issues, Dr. A, a physiotherapist, uses the above soft set approach to identify the subset P that best meets specific physiotherapy-related criteria.

Step 1: Initialize Tracking Structures

1. Create a match tracker: match_tracker = {CL1: 0, CL2: 0, CL3: 0, CL4: 0, CL5: 0, CL6: 0}.
2. Set optimal_element = None.
3. Set max_matches = 0.

Step 2: Evaluate Parameter Matches for Each Clinic

Evaluate how well each clinic matches the criteria in PPP:

1. **Clinic CL1:**
   - **Specialized Equipment:** Satisfied (CL1∈F (specialized equipment)).
   - **Certified Physiotherapists:** Satisfied (CL1∈F(certified physiotherapists)).
   - **Hygiene:** Satisfied (CL1∈F(hygiene)).
   - **Nerve Rehabilitation Programs:** Satisfied (CL1∈F(nerve rehabilitation programs))1`
   - **Post-Treatment Support:** Satisfied (CL1∈F(post-treatment support).
   - **Total Matches = 5.**
   - Update match_tracker: match_tracker = {CL1: 5, CL2: 0, CL3: 0, CL4: 0, CL5: 0, CL6: 0}.
2. **Clinic CL2:**
   - **Specialized Equipment:** Satisfied (CL2∈F(specialized equipment)).
   - **Certified Physiotherapists:** Satisfied (CL2∈F(certified physiotherapists)
   - **Hygiene:** Satisfied (CL2∈F(hygiene)).
   - **Nerve Rehabilitation Programs:** Not satisfied (CL2∉F(nerve rehabilitation programs)).
   - **Post-Treatment Support:** Satisfied (CL2∈F(post-treatment support)).
   - **Total Matches = 4.**
   - Update match_tracker: match_tracker = {CL1: 5, CL2: 4, CL3: 0, CL4: 0, CL5: 0, CL6: 0}.
3. **Clinic CL3:**
   - **Specialized Equipment:** Not satisfied (CL3∉F(specialized equipment)0).
   - **Certified Physiotherapists:** Satisfied (CL3∈F(certified physiotherapists)
   - **Hygiene:** Not satisfied (CL3∉F(hygiene)).
   - **Nerve Rehabilitation Programs:** Satisfied (CL3∈F(nerve rehabilitation programs)).
   - **Post-Treatment Support:** Satisfied (CL3∈F(post-treatment support)).
   - **Total Matches = 3.**
   - Update match_tracker: match_tracker = {CL1: 5, CL2: 4, CL3: 3, CL4: 0, CL5: 0, CL6: 0}.
4. **Clinic CL4:**
   - **Specialized Equipment:** Satisfied (CL4∈F(specialized equipment)).
   - **Certified Physiotherapists:** Not satisfied (CL4∉F(certified physiotherapists)).
   - **Hygiene:** Not satisfied (CL4∉F(hygiene)).

- **Nerve Rehabilitation Programs:** Satisfied (CL4∈F(nerve rehabilitation programs)).
- **Post-Treatment Support:** Satisfied (CL4∈F(post-treatment support)).
- **Total Matches = 3.**
- Update match_tracker: match_tracker = {CL1: 5, CL2: 4, CL3: 3, CL4: 3, CL5: 0, CL6: 0}.

5. **Clinic CL5:**
   - **Specialized Equipment:** Satisfied (CL5∈F(specialized equipment)).
   - **Certified Physiotherapists:** Satisfied (CL5∈F(certified physiotherapists)).
   - **Hygiene:** Not satisfied (CL5∉F(hygiene)).
   - **Nerve Rehabilitation Programs:** Not satisfied (CL5∉F(nerve rehabilitation programs)).
   - **Post-Treatment Support:** Not satisfied (CL5∉F(post-treatment support)).
   - **Total Matches = 2.**
   - Update match_tracker: match_tracker = {CL1: 5, CL2: 4, CL3: 3, CL4: 3, CL5: 2, CL6: 0}.
6. **Clinic CL6:**
   - **Specialized Equipment:** Satisfied (CL6∈F(specialized equipment)).
   - **Certified Physiotherapists:** Not satisfied (CL6∉F(certified physiotherapists)).
   - **Hygiene:** Satisfied (CL6∈F(hygiene)).
   - **Nerve Rehabilitation Programs:** Satisfied (CL6∈F(nerve rehabilitation programs)).
   - **Post-Treatment Support:** Satisfied (CL6∈F(post-treatment support)).
   - **Total Matches = 4.**
   - Update match_tracker: match_tracker = {CL1: 5, CL2: 4, CL3: 3, CL4: 3, CL5: 2, CL6: 4}.

Step 3: Identify Top Matching Element(s)

- **Max Matches:** 5.
- **Best Clinic is:** CL1.

Final Answer:
Clinic CL1 is the best choice for Dr. A based on the given criteria for nerve-related issues.

4.2. Fuzzy Soft Set Theory in Decisions Making

Step 1: Problem Setup

1. **Universal Set (U)**
   U={CL1, CL2,CL3,CL4,CL5,CL6} represents six physiotherapy clinics.
2. **Parameter Set (E)**
   E={specialized equipment, certified physiotherapists, hygiene, patient satisfaction, location, nerve rehabilitation programs, post-treatment support}
3. **Subset of Choice Parameters (P)**
   Dr. A is interested in:
   P={specialized equipment, certified physiotherapists, hygiene, nerve rehabilitation programs, post-treatment support}

4. **Assigned Priorities**
   Dr. A assigns priorities to the parameters:
   - specialized equipment=0.5
   - certified physiotherapists=0.6
   - hygiene=0.4
   - nerve rehabilitation programs=0.7
   - post-treatment support=−0.3

   A choice is said to have been positively influenced by positive values, and negatively by negative values.

Step 2: Fuzzy Soft Set Representation

A fuzzy value represents each clinic's level of adherence to each parameter. Each clinic 's level of satisfaction with the settings is indicated by these numbers, which go from 0 to 1. For example: Clinic CL1 might be rated 0.8 for specialized equipment, 0.7 for certified physiotherapists, and so on. Assume that table 1 below lists the fuzzy values for each clinic and parameter. The table 1 shows the degree to which a clinic satisfies each condition is indicated by each number.

Table 1: The fuzzy soft set's tabular representation (U, E)

Clinic \ Parameter	**Specialized Equipment**	**Certified Physiotherapists**	**Hygiene**	**Nerve Rehab Programs**	**Post-Treatment Support**
CL1	0.8	0.7	0.9	0.8	0.6
CL2	0.7	0.8	0.8	0.7	0.5
CL3	0.6	0.7	0.7	0.6	0.4
CL4	0.8	0.6	0.9	0.9	0.5
CL5	0.7	0.5	0.6	0.6	0.7
CL6	0.9	0.9	0.8	0.9	0.8

Step 3: Calculate Weighted Values

To calculate the priority score for each clinic, we utilize the fuzzy values and priorities for the criteria that are significant to Dr. A. The allocated priority of each parameter is multiplied by its fuzzy value to determine each clinic's priority score. Each membership value should be multiplied by the priority that corresponds to it. For instance, specialist equipment = 0.8×0.5=0.4 for CL1. The table of priorities displayed in table 2

Table 2: Weighted Table

Clinic \ Parameter	**Specialized Equipment**	**Certified Physiotherapists**	**Hygiene**	**Nerve Rehab Programs**	**Post-Treatment Support**	**Total Priority Score**
CL1	0.40	0.42	0.36	0.56	-0.18	1.56
CL2	0.35	0.48	0.32	0.49	-0.15	1.49
CL3	0.30	0.42	0.28	0.42	-0.12	1.3
CL4	0.40	0.36	0.36	0.63	-0.15	1.6
CL5	0.35	0.30	0.24	0.42	-0.21	1.1

CL6	0.45	0.54	0.32	0.63	-0.24	1.7

Step 4: Compute Pairwise Comparisons

Compute the pairwise comparisons with other clinics for each one. After subtracting each clinic's scores for each attribute, add up the outcomes. The pairwise comparison score between clinics Ci and Cj is represented by each cell in the table. This score is determined by adding the variations in the weighted parameter values of the two clinics. For example: Comparison between CL1 and CL2 ($0.4-0.35+0.42-0.48+0.36-0.32+0.56-0.49-(0.18-0.15)=0.07$) . Similarly, all other cells are computed as shown in table 3.

Table 3**:** Comparison table

Clinic	**CL1**	**CL2**	**CL3**	**CL4**	**CL5**	**CL6**
CL1	0	0.07	0.26	-0.04	0.46	-0.14
CL2	-0.07	0	0.19	-0.11	0.39	-0.21
CL3	-0.26	-0.19	0	-0.30	0.20	-0.40
CL4	0.04	0.11	0.30	0	0.50	-0.10
CL5	-0.46	-0.39	-0.20	-0.50	0	-0.60
CL6	0.14	0.21	0.40	0.10	0.60	0

Step 5: Calculate Total Scores

The comparative findings for each other clinic are then added up to determine each competitor's overall score. For example, 0.00+0.07+0.26+(-0.04)+0.46+(-0.14)=0.11 is the total score for Cl1. Table 4 below displays the overall Score table (Decision Table):

Table 4: Total score table

Clinic	**Total Score**
CL1	0.34
CL2	0.72
CL3	−0.15
CL4	0.499
CL5	−0.20
CL6	0.85

Step 6: Select the Best Clinic

CL6 (0.85) is the clinic with the highest overall score. Because Clinic CL6 best meets the requirements, Dr. A should suggest it. Dr. A has the option to choose the next clinic with the greatest score if necessary (CL2, 0.72).

4.3. Fuzzy Soft Set Theory in Group Decision Making

Fuzzy Soft Set Theory addresses imprecision and uncertainty in decision-making, especially in group settings, by compounding fuzzy and soft sets. It settles disputes, combines differing viewpoints, depicts ambiguous desires, and methodically ranks options. It successfully manages subjective and partial information by integrating degrees of membership. This idea is significant because it fosters consensus among decision-makers, supports complicated criteria, improves decision accuracy, and takes into account a variety of viewpoints. Its practical uses in the fields of public policy, commerce, and healthcare make it an effective instrument for group decision-making in the face of ambiguity, guaranteeing impartial and trustworthy results in intricate, multi-criteria situations. The following is an algorithm for group decisions utilizing fuzzy soft set theory:

Algorithm 4.1

Step 1: Priorities of Input

The panel of judges (there are n judges total) should provide the priority values P=(P1,P2,…,Pn) for each parameter.

Step 2: Process Each Judge's Contributions

For every judge (Ji, where i=1,2,…,n), complete the subsequent steps:

a. Adding the fuzzy soft set:
- The fuzzy soft set F_E of the judge ought to be noted and arranged in a table

b. Make the Priority Table (PT):
- Through the multiplication of each parameter's fuzzy value by its associated priority.
- The weighted values in each row are added to determine the overall score.

c. Make the Comparison Table (CT):
- Via subtracting the total score of every row from the score of every other row.

d. Determine the Row Scores in CT:
- To ascertain each conetder's score, use the comparison table's row total.

e. Create the Decision Table:
- By sorting the rows based on their CT row scores,
- choosing the row with the highest score.

Step 3: Combine the Judges Ranking:

Combine each judge's rankings to get a consolidated rank table.

Step 4: Pick the Best Contender:

a. Find Rank Sums:
- The overall rank sum for every row may be found using the rank table.

b. Select the Best Option:
- The contender with the lowest rank sum is the best option .
- In the event of a tie, consider the contender who received the highest score in the most crucial parameter.

c. Keep going until the ultimate ranks are determined:
- Continue severing connections as needed to finish the ranks.

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The criteria for choosing clinic are E= {specialized equipment, certified physiotherapists, hygiene, nerve rehabilitation programs, post-treatment support}. Let U be a collection of six clinics, {Cl1,Cl2,Cl3,Cl4,Cl5,Cl6}.Think about FSS(U,E) explaining the "clinic selection." Keep in mind that {J1, J2, J3} are the judges who score each clinic according to their performance after evaluating based on a set of specified criteria. {e1, e2, e3, e4, e5} can be used to represent the parameters {specialized equipment, certified physiotherapists, hygiene, nerve rehabilitation programs, post-treatment support}. The following steps outline the solution for the given physiotherapy clinic selection problem using the above algorithm.

Step-1: Input the Priority of Judges

Suppose we have n=3 judges, and each judge assigns a priority rating to the parameters in P={certified physiotherapists, specialized equipment, hygiene, nerve rehabilitation programs, post-treatment support}. Assuming three judges and their priorities for each parameter given in table5, let's assign the following priorities:

Judge 1: P1=(3,2,5,4,1)
Judge 2: P2=(4,3,5,2,1)
Judge 3: P3=(2,3,4,5,1)

Table 5: Priority rank table given by Judges

Parameter	**Judge 1 Priority**	**Judge 2 Priority**	**Judge 3 Priority**	**Total Priority**	**Parameter Rank**
Specialized Equipment (SE)	3	4	2	9	3
Certified Physiotherapists (CP)	2	3	3	8	4
Hygiene (HYG)	5	5	4	14	1
Nerve Rehabilitation Programs (NRP)	4	2	5	11	2
Post-Treatment Support (PTS)	1	1	1	3	5

Step 2: Process Each Judge's Contributions

Do the following for every judge Ji(i=1,2,3).

2(a) First, enter the fuzzy soft set Provided by each Judge J1

Each judge gives each clinic a fuzzy soft set. The fuzzy soft set is a matrix in which each item denotes a clinic's degree of fulfilment or level of satisfaction with regard to a specific criterion. Assume that each judge has a fuzzy soft set that includes each clinic's level of satisfaction with the settings in P shown in table 6. A fuzzy value is represented by each item in the matrix; for instance, 0.7 denotes moderate fulfilment and 1 denotes full pleasure.

Table 6: Fuzzy Soft Set (FS) of Judge 1

Clinic	**SE**	**CP**	**HYG**	**NRP**	**PTS**
CL1	0.9	0.8	1.0	0.7	0.8
CL2	0.7	0.9	0.8	1.0	0.6
CL3	1.0	0.7	0.9	0.8	1.0
CL4	0.8	0.9	0.9	0.6	0.7
CL5	0.6	0.8	0.7	0.9	0.8
CL6	0.7	0.7	0.8	0.8	0.9

2(b). Make the Priority Table (PT) for judge J1

By taking each parameter's fuzzy value and multiplying it by the set priority. The total score for every row is determined by adding the weighted values in the row. The determined priority table (PT) for Judge 1 is displayed in Table 7 below:

Table 7 :Priority Table (PT) of Judge 1

Clinic	**SE**	**CP**	**HYG**	**NRP**	**PTS**	**Sum of Row**
CL1	0.9×3=2.7	0.8×2=1.6	1.0×5=5.0	0.7×4=2.8	0.8×1=0.8	13.9

CL2	0.7×3=2.1	0.9×2=1.8	0.8×5=4.0	1.0×4=4.0	0.6×1=0.6	12.5
CL3	1.0×3=3.0	0.7×2=1.4	0.9×5=4.5	0.8×4=3.2	1.0×1=1.0	13.1
CL4	0.8×3=2.4	0.9×2=1.8	0.9×5=4.5	0.6×4=2.4	0.7×1=0.7	11.8
CL5	0.6×3=1.8	0.8×2=1.6	0.7×5=3.5	0.9×4=3.6	0.8×1=0.8	11.3
CL6	0.7×3=2.1	0.7×2=1.4	0.8×5=4.0	0.8×4=3.2	0.9×1=0.9	11.6

2(c). Make the Comparison Table (CT) for judge 1:

To create the comparison table, subtract the total score of each row from that of the previous row, for instance, 13.9 - 12.5 = 1.4. This discrepancy demonstrates how much Judge 1 thinks CL1 is better than CL2. Table 8 displays the comparison table that is created when this process is repeated for each pair of rows. Additionally, depending on Judge 1 priority, it provides the clinic overall score and ranking (steps 2(d) and 2(e) of the provided procedure).

Table 8: Comparison Table (CT) for Judge 1

Clinic	**CL1**	**CL2**	**CL3**	**CL4**	**CL5**	**CL6**	**Score**	**Rank**
CL1	0	1.4	0.8	2.1	2.6	2.3	9.2	1
CL2	−1.4	0	−0.4	0.7	1.2	1.0	2.9	3
CL3	−0.8	0.4	0	1.3	1.8	1.5	5	2
CL4	−2.1	−0.7	−1.3	0	0.5	0.2	0.7	4
CL5	−2.6	−1.2	−1.8	−0.5	0	−0.3	0	6
CL6	−2.3	−1.0	−1.5	−0.2	0.3	0	0.3	5

The Judge 2 fuzzy set, Judge 2 priority table, and Judge 2 comparison table are displayed in Tables 9, 10, and 11, respectively, after steps 2(a), 2(b), and 2(c) are repeated for Judge 2.

Table 9: Fuzzy Soft Set (FS) of Judge 2

Clinic	**SE**	**CP**	**HYG**	**NRP**	**PTS**
CL1	0.8	0.9	1.0	0.8	0.9
CL2	0.6	0.8	0.7	1.0	0.7
CL3	1.0	0.7	0.8	0.9	0.8
CL4	0.9	0.9	0.8	0.6	0.7
CL5	0.7	0.7	0.8	0.9	0.8
CL6	0.8	0.8	0.9	0.8	0.7

Table 10: Priority Table (PT) of Judge 2

Clinic	**SE**	**CP**	**HYG**	**NRP**	**PTS**	**Sum of Row**
CL1	0.8×4=3.2	0.9×3=2.7	1.0×5=5.0	0.8×2=1.6	0.9×1=0.9	13.4
CL2	0.6×4=2.4	0.8×3=2.4	0.7×5=3.5	1.0×2=2.0	0.7×1=0.7	10.0
CL3	1.0×4=4.0	0.7×3=2.1	0.8×5=4.0	0.9×2=1.8	0.8×1=0.8	12.7
CL4	0.9×4=3.6	0.9×3=2.7	0.9×5=4.5	0.6×2=1.2	0.7×1=0.7	12.2
CL5	0.7×4=2.8	0.7×3=2.1	0.8×5=4.0	0.9×2=1.8	0.8×1=0.8	11.5
CL6	0.8×4=3.2	0.8×3=2.4	0.9×5=4.5	0.8×2=1.6	0.7×1=0.7	12.4

Table11 : Comparison Table (CT) for Judge 2

Clinic	CL1	CL2	CL3	CL4	CL5	CL6	Score	Rank
CL1	0	3.4	0.7	1.2	1.9	1.0	8.2	1
CL2	−3.4	0	−2.7	−2.2	−1.5	−2.4	0	6
CL3	−0.7	2.7	0	0.5	0.2	−0.3	3.4	3
CL4	−1.2	2.2	−0.5	0	−0.3	−0.7	2.2	4
CL5	−1.9	1.5	−0.2	0.3	0	−0.3	1.8	5
CL6	−1.0	2.4	0.3	0.7	0.3	0	3.7	2

The Judge 3 fuzzy set, Judge 3 priority table, and Judge 3 comparison table are displayed in Tables 12, 13, and 14, respectively, after steps 2(a), 2(b), and 2(c) are repeated for Judge 2.

Table 12: Fuzzy Soft Set (FS) of Judge 3

Clinic	SE	CP	HYG	NRP	PTS
CL1	1.0	0.9	0.9	0.7	0.9
CL2	0.8	0.8	0.7	1.0	0.7
CL3	0.9	0.8	1.0	0.8	1.0
CL4	0.7	1.0	0.8	0.9	0.8
CL5	0.9	0.7	0.9	1.0	0.9
CL6	0.8	0.9	0.8	0.9	0.7

Table 13 :Priority Table (PT) of Judge 3

Clinic	SE	CP	HYG	NRP	PTS	Sum of Row
CL1	1.0×2=2.0	0.9×3=2.7	0.9×4=3.6	0.7×5=3.5	0.9×1=0.9	12.7
CL2	0.8×2=1.6	0.8×3=2.4	0.7×4=2.8	1.0×5=5.0	0.7×1=0.7	12.5
CL3	0.9×2=1.8	0.8×3=2.4	1.0×4=4.0	0.8×5=4.0	1.0×1=1.0	13.2
CL4	0.7×2=1.4	1.0×3=3.0	0.8×4=3.2	0.9×5=4.5	0.8×1=0.8	12.9
CL5	0.9×2=1.8	0.7×3=2.1	0.9×4=3.6	1.0×5=5.0	0.9×1=0.9	13.4
CL6	0.8×2=1.6	0.9×3=2.7	0.8×4=3.2	0.9×5=4.5	0.7×1=0.7	12.7

Table 14: Comparison Table (CT) for Judge 3

Clinic	CL1	CL2	CL3	CL4	CL5	CL6	Score	Rank
CL1	0	1.4	−0.5	−0.7	−0.7	0	1.4	4
CL2	−1.4	0	−1.2	−0.6	−0.9	−1.4	0	5
CL3	0.5	1.2	0	−0.3	−0.2	0.5	2.2	2
CL4	0.7	0.6	0.3	0	−0.3	0.3	1.9	3
CL5	0.7	0.9	0.2	0.3	0	0.3	2.4	1
CL6	0	1.4	−0.5	−0.3	−0.3	0	1.4	4

Step 3: Combine the Judges Ranking:

Combine each judge's rankings to get a consolidated rank table as shown in table 15.

Table 15: Consolidated rank table

Clinic	Rank (Judge 1)	Rank (Judge 2)	Rank (Judge 3)
CL1	1	1	4
CL2	3	6	5
CL3	2	3	2
CL4	4	4	3
CL5	6	5	1
CL6	5	2	4

Step 4: Pick the Best Contender

4(a). Find Rank Sums:

Determine the overall rank sum for each row in table 16 using the rank table.

Table 16: Rank Sum

Clinic	Rank Sum	Rank
CL1	6	1
CL2	14	5
CL3	7	2
CL4	11	3
CL5	12	4
CL6	11	3

4(b). Select the Best Option

The clinic with the lowest rank sum is the best option. However, both clinics (CL4 and CL6) have the same rank, which is 3. The clinic with the highest absolute priority column can be found in the rank table above. In this instance, "Hygiene" is the option with the highest importance. The amended rank table may be found below in Table 17.

Table 17: Modified Rank table

Clinic	Hygiene (J1)	Hygiene (J2)	Hygiene (J3)	Sum	Rank
CL4	4.5	4.5	3.2	12.7	3
CL6	4.0	4.5	3.2	11.7	4

If we update this new rating in the rank table, clinics CL5 and CL6 will once again have the same rank, which is 4. This results in the identification of the clinic with the highest absolute priority column. In this instance, "Hygiene" is the option with the highest importance. The updated rank table is shown in the table 18 below.

Table 18: Modified Rank table

Clinic	Hygiene (J1)	Hygiene (J2)	Hygiene (J3)	Sum	Rank
CL5	3.5	4.0	3.6	11.1	5
CL6	4.0	4.5	3.2	11.7	4

If we update this new rating in the rank table, clinics CL2 and CL5 will once again have the same rank, which is 5. As a result, we need to modify the rank and follow the previously described procedure. The updated rank table is shown in the table 19 below.

Table 19: Modified Rank table

Clinic	Hygiene (J1)	Hygiene (J2)	Hygiene(J3)	Sum	Rank
CL2	4.0	3.5	2.8	10.3	6
CL5	3.5	4.0	3.6	11.1	5

After repeating the previously indicated procedure, the final rank table is given in the table 20 below.

Table 20: Final Rank

Clinic	Rank
CL1	1
CL2	6
CL3	2
CL4	3
CL5	5
CL6	4

The judges' panel has determined that contender CL1 is the best option based on the final rank table 20 above.

5. Result and Discussion

Using fuzzy soft set theory and soft set theory to make decisions demonstrates how effective they are at handling imprecision and uncertainty. In direction to determine the top clinic for the clinic selection problem, soft set theory successfully decreased the number of alternatives using a parameter satisfaction method by evaluating the extent to which specific criteria were met. CL1 was the top clinic, meeting all the standards and offering further improvement through tie-breaking techniques. The fuzzy soft set approach enhanced this process by adding priority weights for each criterion and levels of parameter satisfaction. By comparing clinics and computing priority scores, this method provided a more thorough evaluation. Table 4 indicates that clinic CL6 was the best one based on its greatest priority score. This idea was extended to group decision-making by the fuzzy soft set technique, which ranked applications by combining inputs from several judges and creating priority and comparison tables. By employing weights that represented the priorities of the agreement, this method allowed for a range of opinions while determining which clinic was best. These results demonstrate the accuracy, flexibility, and transparency of the soft and fuzzy soft set approaches, which make them valuable tools for difficult decision-making scenarios in a variety of fields.

The soft set technique found that clinic Cl1 is appropriate since it relied on binary parameter satisfaction without considering priority or intensity. By integrating degrees of satisfaction and parameter priority, the fuzzy soft set method improved evaluation and ranked CL6 as the best clinic based on weighted criteria, addressing nuances missed by the binary approach. The group decision-making extension significantly enhanced the process by effectively balancing individual biases and consensus by merging different viewpoints with fuzzy weights. While the soft set strategy is easier, the fuzzy and group decision-making approaches provide more accuracy and adaptability for complex scenarios.

6. Conclusion

The use of soft sets and fuzzy soft sets in collective decision-making under uncertainty is the main topic of this research, which also discusses the relative benefits of both. Though its simplicity restricts its capacity to manage complicated priorities and intensities, soft set theory offers a straightforward framework for narrowing choices by meeting binary characteristics. These drawbacks are addressed by fuzzy soft sets, which incorporate parameter preferences and satisfaction levels to allow for more accurate evaluations, including clinic assessments. Fuzzy soft sets perform even better in group decision-making when different viewpoints are integrated and balanced consensus is reached through the use of aggregated fuzzy weights. The results show that fuzzy soft sets and their extensions for collective decision-making are more accurate, flexible, and reliable, making them more appropriate for intricate, cooperative situations. Soft sets, meanwhile, continue to work well for simple choices. The peculiarity of this study is that it directly compares fuzzy soft sets, soft sets, and their collective decision-making applications, demonstrating how well they manage uncertainty and enhance decision outcomes in a single framework, particularly in complex and collaborative circumstances.

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